A New Approach to Wrinkling Prediction of Space Membrane Structures

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Abstract

Thin-film membranes are basic elements of a variety of space structures, such as the Next Generation Space Telescope (NGST), the Inflatable Synthetic Aperture Radar (ISAR), solar arrays, solar sails, and reflectors [1, 2]. Thin-film membranes usually have little compression resistance capability, and hence wrinkle when negative stresses appear. Wrinkling, which is a local buckling effect, degrades the performance and reliability of space membrane structures, and can cause malfunction and even damage of these systems. Therefore, good understanding and accurate prediction of wrinkling formation and wrinkled patterns of membrane structures is important to the development of many space structures.

This paper presents an innovative method, called the Bar-Networking Approach (BNA), for modeling and analysis of partially wrinkled membranes. In the BNA, a membrane is modeled as a network of bars with a special nonlinear constitutive relation characterizing wrinkling behaviors of the membrane. Based on a parametric variational principle (PVP) and an optimization procedure, accurate solutions for the deformations and stress distributions of the wrinkled membrane are obtained efficiently without the need for conventional numerical iterations.

Basic Ideas and Principles

The BNA-based modeling and analysis takes the following three major steps.

Step 1. Establishment of a wrinkled bar model

For a bar in the network, a modified strain-stress relation is

$$\varepsilon = \frac{\sigma}{E} - \lambda; \quad \lambda \geq 0 \qquad (1)$$

where ε and σ are the strain and stress of the bar, respectively, and E is Young's modulus. The parameter λ is a variable controlling parameter, which physically represents a compensated amount of strain due to wrinkling. Now define a constraint function

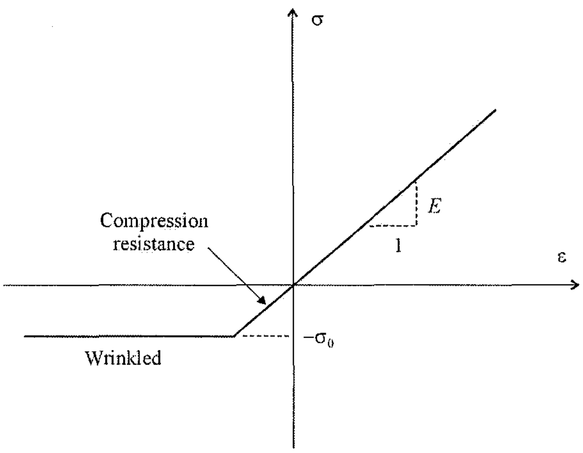


Fig. 1

$$f(\varepsilon, \lambda) = E\varepsilon + \sigma_0 + E\lambda \tag{2}$$

where σ_0 is a stress parameter describing small or zero compression resistance of the bar (see Fig. 1 for instance). With λ and f, it can be shown that two states of the bar network are determined by the following constraint conditions

(S1) Unwrinkled state: $\lambda = 0$ and $f \geq 0$ (3a)

(S2) Wrinkled state: $\lambda \geq 0$ and $f = 0$ (3b)

Equation (1) with conditions (3) gives a nonlinear constitutive relation that can be used to systematically describe wrinkled and unwrinkled states of thin-film membranes.

Step 2. Development of a parametric variational principle

With the controlling parameter λ introduced in Step 1, the total potential energy of a bar network is derived as follows

$$\Pi_\lambda = \frac{1}{2}\{\hat{u}\}^T[K]\{\hat{u}\} - \{\hat{u}\}^T\{\hat{p}\} + \{\hat{u}\}^T[G(\lambda)]\{\lambda\} \tag{4}$$

where $\{\hat{u}\}$ is a global displacement vector, $\{\hat{p}\}$ is a global load vector, $\{\lambda\}$ is a vector containing the controlling parameters of all bars. The variation of the potential energy, $\delta\Pi_\lambda = 0$, leads to the governing equations and constraint conditions for the bar network. This eventually leads to a finite element formulation.

The Π_λ is different from conventional Minimum Potential Energy Principle (MPEP) in two major aspects. First, the controlling parameters $\{\lambda\}$, which are related to strain quantities, do NOT participate in the variation of Π_λ [3]. For this reason, Π_λ is called the Parametric Variational Principle (PVP). Second, it can be shown that Π_λ reaches its *global* minimum at the exact solution. On the other hand, the minimum energy property of MPEP would be destroyed if conditions for wrinkled state like (3b) are introduced.

Step 3. Solution by Mathematical Programming

With the global minimum property of the proposed PVP, solution of the original membrane is converted to that of the following optimization problem:

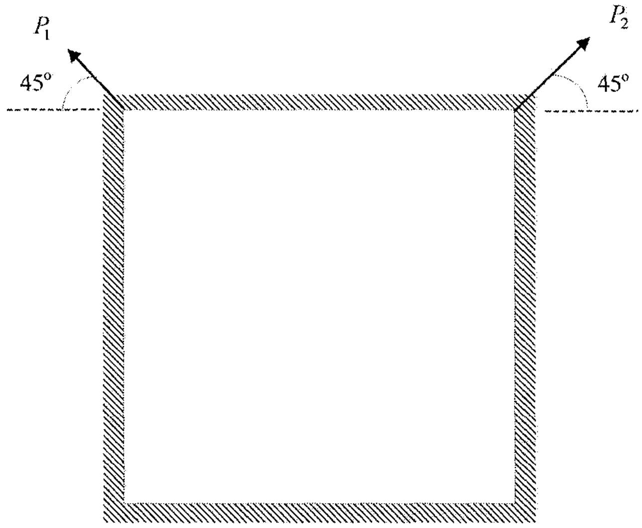
Minimize Π_λ (5)

Subject to the constraints given in (3).

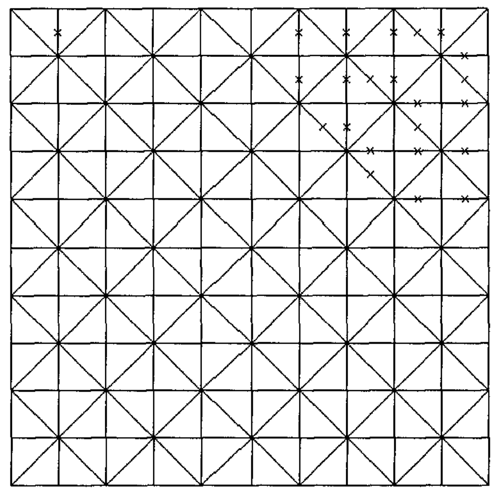
This problem can be solved by several optimization methods. For the current investigation, the non-interior continuation method [4] is efficient.

An Illustrative Example of Wrinkling Formation

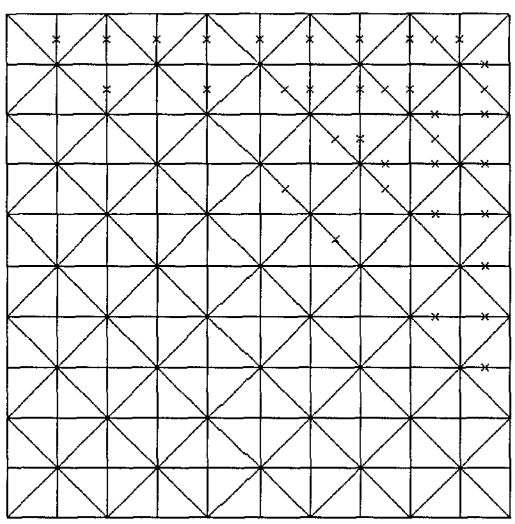
In Fig. 2(a), a square membrane hinged at the boundary is stretched by two corner forces P_1 and P_2. Let $P_1 = 1$, and P_2 be adjustable. The membrane is modeled by a network of 320 bars with small compression resistance. By the proposed BNA, the formation and growth of wrinkles in the membrane as P_2 increases is predicted, and presented in Figs. 2(b) to 2(d), where bars in wrinkled state (S2) are marked by cross ($\times$). The wrinkle longitudinal direction can be identified by lines connecting adjacent crosses. Note that the growth of wrinkles is not symmetric due to the asymmetry of the external forces.



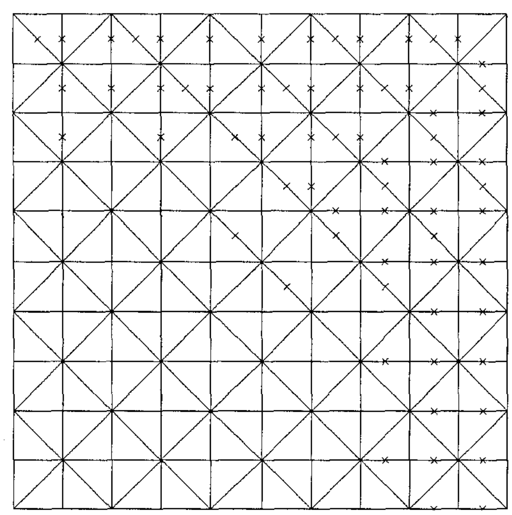
(a)



(b) $P_2 = 6$



(c) $P_2 = 9$



(d) $P_2 = 20$

Fig 2. Formation of wrinkled pattern of a square membrane ($E = 100$, $\sigma_0 = 30$)

Special Features and Capabilities

The proposed Bar Networking Approach has several special features and capabilities.

(i) The BNA, besides being able to predict wrinkled patterns, is capable of portraying the formation and expansion of wrinkled regions of membranes as external loads increase. This feature provides a useful aid for understanding the mechanism of wrinkles and mechanics of partially wrinkled membrane structures.

(ii) The BNA with PVP is convenient for finite element formulation, which makes it practical and versatile in modeling complex membrane structures.

(iii) The global minimum of the parametric potential energy Π_λ guarantees fast convergence of solutions. In the numerical example, exact solutions are obtained in a finite number of steps in computation. Unlike conventional methods, the BNA does not need iterations of stresses, which sometimes cause convergence problems in conventional analysis.

In short, the proposed BNA is a useful tool for development of space inflatable structures. The experimental validation of the bar-networking model of membranes is currently underway. In the meantime, the PVP concept is being extended to two-dimensional membrane models.

References

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